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Nonlinear Stabilizing Control of High Angle of Attack Flight Dynamics

by

E. H. Abed

SYSTEMS RESEARCH CENTER

UNIVERSITY OF MARYLAND

COLLEGE PARK, MARYLAND 20742

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Eyad H. Abed

Department of Electrical Engineering
and the Systems Research Center
University of Maryland
College Park, MD 20742

1. Introduction. This paper discusses a new approach to the feedback control of aircraft at high angles of attack. This approach is based on recent results on control of nonlinear systems at bifurcation points [3, 4, 5] . The lateral dynamics of a slender aircraft studied by Ross [6] provides a convenient model for illustrating this application. It is shown how local bifurcation control can be used in the stabilization of the so-called "wing rock" phenomenon. The appropriateness of this approach derives from the fact that unstable motions at high incidence, such as stall, spin entry, wing rock and roll coupling, have been attributed to bifurcations in the vehicle dynamic equations. The instability can indeed be predicted via bifurcation analysis, and depends critically on the nonlinear aerodynamic force modeling. Bifurcation control results can facilitate the synthesis of nonlinear stabilizing feedback controls for aircraft at the onset of vortex separation; an angle of attack limiter is not employed in this approach. Possible means of achieving the required control laws include the control surface deflections and thrust vector control.

The bifurcation control technique considered here, developed in [3, 4] , provides an analytically-based, algorithmic approach to the stabilization of vehicles at the onset of flow separation. In classical terminology, the goal of the nonlinear control design is to render the stability boundary at high incidence "safe" (as opposed to "dangerous"). Practically the result would be to enlarge the flight envelope, by ensuring that any near-stall sustained deviation from trim is stable and of small amplitude, and therefore tolerable. Divergence cannot result from such a state without the introduction of a large disturbance. Thus, pre- and post-stall dynamics are stabilized.

The remainder of this paper is organized as follows. Section 2 presents a brief discussion of relevant background material on high angle of attack dynamic modeling and associated nonlinear effects. In Section 3, the bifurcation control framework of [3, 4] is reviewed. In Section 4, application of the bifurcation control technique to a model for the high incidence dynamics of the Handley Page 115 research aircraft model of Ross [6] is presented.

2. High Angle of Attack Flight Dynamics. Analytical study of rigid body flight at high angle of attack (α) is made difficult by essentially three main factors:

(i) *Strong dynamic coupling of longitudinal and lateral-directional motions* almost always necessitates the use of a six degree-of-freedom model, with state variables representing angular orientation and rates, as well as translational position and velocity;

(ii) *Nonlinearity of the dynamic motion equations.* Since the trim condition borders on instability in the high α regime, linearized equations of motion are of little use in predicting dynamic behavior; and

(iii) *Nonlinear dependence of aerodynamic forces on states.* Vortex shedding and flow separation complicate aerodynamic modeling in this flight regime, especially with regard to wing-body and tail-body interference effects. In the supersonic range, time-history effects also become important.

Although nonlinear aerodynamic modeling is difficult and highly configuration-dependent, the dynamics of rigid body flight are well understood. Viewing an aircraft as a rigid body of fixed mass, its motion is completely determined (once the aerodynamic forces are known) by the following six equations, which are expressed in the basic Euler form with respect to body axes:

$$\dot{u} = \frac{f_x}{m} + \frac{X}{m} + vr - wq \tag{2.1}$$

$$\dot{v} = \frac{f_y}{m} + \frac{Y}{m} + wp - ur \tag{2.2}$$

$$\dot{w} = \frac{f_z}{m} + \frac{Z}{m} + qu - vp \tag{2.3}$$

$$\dot{p} = \frac{L}{I_x} - \left(\frac{I_z - I_y}{I_x}\right)qr + \frac{I_{xz}}{I_x}(\dot{r} + pq) + \frac{I_{xy}}{I_x}(\dot{q} - rp) + \frac{I_{yz}}{I_x}(q^2 - r^2) \tag{2.4}$$

$$\dot{q} = \frac{M}{I_y} + \left(\frac{I_x - I_z}{I_y}\right)pr - \frac{I_{xz}}{I_y}(p^2 - r^2) + \frac{I_{xy}}{I_y}(\dot{p} + qr) + \frac{I_{yz}}{I_y}(\dot{r} - pq) \tag{2.5}$$

$$\dot{r} = \frac{N}{I_z} - \left(\frac{I_y - I_x}{I_z}\right)qp + \frac{I_{xz}}{I_z}(\dot{p} - qr) + \frac{I_{xy}}{I_z}(p^2 - q^2) + \frac{I_{yz}}{I_z}(\dot{q} + rp). \tag{2.6}$$

Here u, v, w are the velocity components of the center of gravity along the x, y and z axes, respectively; p, q, r are the components of the angular velocity about these axes; m is the mass of the aircraft; f_x, f_y and f_z are the Eulerian components of the instantaneous gravity force; X, Y and Z are the total aerodynamic forces; and L, M, N are the total moments with respect to the standard (body) axes. The notation I_x, I_{xy}, etc. refers to the moments and products of inertia relative to the center of mass.

If the mass distribution of the aircraft is symmetrical with respect to, say, the xz plane, then $I_{xy} = I_{yz} = 0$ and the last two terms in each of the equations (2.4)-(2.6) vanish. This is a very good approximation for most aircraft in steady flight, but may not be appropriate if, for example, a significant amount of fuel motion occurs in a tank ("fuel sloshing").

The aerodynamic forces and moments acting on the aircraft are expressed in terms of the familiar dimensionless aerodynamic coefficients:

$$X = (1/2)\, C_x \rho V^2 S\,,\; Y = (1/2)\, C_y \rho V^2 S\,,\; Z = (1/2)\, C_z \rho V^2 S\,, \tag{2.7a}$$

$$L = (1/2)\, C_l \rho V^2 S b\,,\; M = (1/2)\, C_m \rho V^2 S c\,,\; N = (1/2)\, C_n \rho V^2 S b\,; \tag{2.7b}$$

with C_x, C_l, etc. being the dimensionless coefficients; ρ the atmospheric density; S the reference area; c the mean aerodynamic chord; b the wing span; and V the magnitude of the velocity of the center of gravity. The aerodynamic coefficients C_x, C_y, etc. above are not constants but depend on the orientation and angular velocity of the vehicle. Accurate analytical evaluation of these coefficients is difficult and data from wind tunnel tests is usually relied upon (when available). Note that the effect of control surface deflections is incorporated into the model through the aerodynamic coefficients. The control surface deflections correspond to the values of the aileron, rudder and elevator angles δ_a, δ_r and δ_e, respectively. The thrust may also be viewed as a control variable, such as in thrust vector control.

Equations (2.1)-(2.3) may be recast in terms of the angle of attack α, the sideslip angle β and the velocity magnitude V, where

$$\alpha = \tan^{-1}\frac{w}{u},\; \beta = \sin^{-1}\frac{v}{V},\; \text{and } V = (u^2 + v^2 + w^2)^{1/2}\,. \tag{2.8}$$

This is facilitated by expressing the aerodynamic forces X, Y, Z as

$$\begin{aligned} X &= L\sin\alpha - D\cos\beta\cos\alpha \\ Y &= D\sin\beta \\ Z &= -L\cos\alpha - D\cos\beta\sin\alpha \end{aligned} \tag{2.9}$$

where L and D are the net lift and drag components of the aerodynamic force:

$$L = (1/2)\, C_L \rho V^2 S = \text{lift}$$

$$D = (1/2)\, C_D \rho V^2 S = \text{drag}$$

The transformed equations (for a symmetrical body) may be found in Adams [7] .

It appears that the first analytical study of bifurcation or jump phenomena in high α flight dynamics was due to Schy and Hannah [8] although predictions of the utility of the approach had previously been made (Phillips [9] , Pinsker [10] , Rhoads and Schuler [11]). This analysis benefited from the report of Adams [7] which used constrained minimization techniques to solve the nonlinear algebraic equations for an equilibrium spin. In [8] it was shown how the so-called "pseudosteady-state solutions" could be obtained by solving a single polynomial in roll rate with coefficients that were functions of the control inputs. This was followed by the paper Young, Schy and Johnson [12] in which the inclusion of aerodynamics nonlinear in the angle of attack led to the solution of two polynomials in roll rate whose coefficients are functions of angle of attack and the control inputs. Analytical results were compared with calculated time histories to demonstrate the validity of the method for predicting jump-like instabilities. This work

Only the linear, quadratic and cubic terms in an applied feedback $u(x)$ have potential for influencing the stability. Stability may be checked by computing so-called *bifurcation coefficients.* Public domain software packages exist which perform the bifurcation analysis, as well as follow the bifurcation branches in one or two parameters. We mention, in particular, the BACTM software of Carroll and Mehra [13] , the package BIFOR2 of B. Hassard [16] , SUNY Buffalo, and the interactive package AUTO [17] authored by E. Doedel at Concordia University, Montreal. Bifurcation analysis software can be used, in conjunction with simulation tools, to obtain a global picture of the efficacy of a particular control law design.

Theorem. *Let the uncontrolled system* ((3.3) with $u = 0$) *undergo an unstable Hopf bifurcation to periodic solutions. Then there is a smooth feedback* $u(x)$ *with* $u(0) = 0$ *which solves the local Hopf bifurcation stabilization problem for Eq.* (3.3) *provided that*

$$0 \neq \mathrm{Re}\ \{ -2lQ_0(r, \tfrac{1}{2}L_0^{-1}\gamma) + lQ_0(\bar{r}, \tfrac{1}{2}(2i\omega_c I - L_0)^{-1}\gamma) + \tfrac{1}{4}l[\,2L_1 r + L_1\bar{r}\,]\ \}. \tag{3.7}$$

A whole family of conditions similar to Eq. (3.7) may be derived, each corresponding to a particular family of nonlinear stabilizing control laws $u(x)$. The results are available for both static and Hopf bifurcations [18] . Eq. (3.7) indicates the relative computational ease with which the results may be applied; only a Taylor series expansion of the dynamic equations and standard spectral calculations are needed.

4. Wing Rock Stabilization for the HP115. Nonlinear feedback control laws ensuring a "safe" stability boundary at high angle of attack may be designed using the bifurcation control results discussed in Section 3. Use of this approach has several advantages over most standard approaches to high angle of attack flight control: (i) Rather than avoiding stall through angle of attack limiters, the aim is to use feedback to permit stable flight in the stall and post-stall regime; (ii) All nonlinear stabilizing feedback controls can be parametrized, allowing consideration of other criteria besides local asymptotic stability, such as size of the domain of stability. (The theory provides a way to estimate the achieved domain of stability.); and (iii) Stall stabilization may well be preferable to deferring stall to higher angles of attack, since, as noted by Babister [2] , the latter often leads to a much more violent stall at a higher angle of attack.

To indicate the type of aircraft control problem which can benefit directly from the bifurcation control approach, we mention thrust vector control as a means for stabilizing aircraft longitudinal and/or lateral dynamics at high incidence. We note that in a recent paper [19] , Ashley has concluded that thrust vector control will be crucial in subsonic maneuvering at extreme angles of attack.

Consider the following model [6, 20] for the lateral dynamics of the HP115 aircraft

laws for a control system

$$\dot{x} = f_\mu(x, u). \tag{3.3}$$

In (3.3), u is a control function and μ is, as before, a bifurcation parameter. The main assumption of the approach is simply that the 'uncontrolled' system (obtained by setting $u \equiv 0$ in (3.3))

$$\dot{x} = f_\mu(x, 0) \tag{3.4}$$

undergoes a bifurcation as μ is varied. The most important benefit of the approach is an algorithmic design aid which generates, in a computationally efficient way, all the purely nonlinear feedback control laws $u = u(x)$ ensuring stability of the new ("bifurcated") solutions for the controlled system. In the case of a Hopf bifurcation, for instance, one can design feedbacks ensuring that for any value of the parameter μ in some domain around the critical value, the state converges either to the original equilibrium solution or to a nearby (stable) periodic solution of small (nearly zero) amplitude. Since the amplitude of the bifurcated periodic orbits is small for small μ, this type of (stable) oscillatory behavior may be tolerable from a practical point of view, and is certainly preferable to divergence or uncontrolled wing rock.

Our approach to the local feedback stabilization problem has the novel feature that it facilitates the derivation of generally valid analytical criteria for stabilizability, as well as specific stabilizing feedback controls. This is possible through use of bifurcation formulae which involve only Taylor series expansion of the vector field and eigenvector computations. To illustrate the generality of the approach and the simplicity of the associated calculations, a representative result is given next.

Consider a one-parameter family of nonlinear control systems (3.3) where $x \in R^n$, u is a scalar control, μ is a real-valued parameter, and the vector field f_μ is sufficiently smooth. Suppose that for $u \equiv 0$, Eq. (3.3) has an equilibrium point $x_0(\mu)$ which depends smoothly on μ. The linearization of Eq. (3.3) at $x = 0$, $u = \mu = 0$ is given by

$$\dot{x} = Ax + bu \tag{3.5}$$

where $A := \frac{\partial f}{\partial x}(0,0)$ and $b := \frac{\partial f}{\partial u}(0,0)$. The nature of the stabilizing nonlinear feedack controls given by the theory depends on the controllability properties of this linear system. Rewrite Eq. (3.3) in the series form

$$\begin{aligned} \dot{x} = {} & L_0 x + u\gamma + uL_1 x + Q_0(x,x) \\ & + C_0(x,x,x) + \cdots \end{aligned} \tag{3.6}$$

where the terms not written explicitly are of higher order in x, u and μ than those which are. Thus L_0 and L_1 are square matrices, γ is a constant vector, $Q_0(x,x)$ is a quadratic form generated by a symmetric bilinear form $Q_0(x,y)$ giving the second order (in x) terms at $u = 0$, $\mu = 0$, and $C_0(x,x,x)$ is a cubic form generated by a symmetric trilinear form $C_0(x,y,z)$ giving the third order (in x) terms at $u = 0$, $\mu = 0$. (Note that L_0 is simply A of Eq. (3.5), and γ corresponds to b.) Denote by r the right (column) and by l the left (row) eigenvector of L_0 with eigenvalue $i\omega_c$. Normalize by setting the first component of r to 1 and then choose l so that $lr = 1$.

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at high incidence:

$$\dot{\beta} = \sin\alpha\, p - \cos\alpha\, r + \frac{g\cos\alpha}{V}\phi \tag{4.1a}$$

$$+ \frac{\rho S V}{m}\{ y_1\beta + y_3\beta^3 + y_p\frac{ps}{V} + y_r\frac{rs}{V} \}$$

$$\dot{p} = \frac{\rho S V^2 s I_z}{I_x I_z - I_{xz}^2}\{ l_1\beta + l_3\beta^3 + l_p\frac{ps}{V} + l_r\frac{rs}{V} \tag{4.1b}$$

$$+ \frac{I_{xz}}{I_z}(n_1\beta + n_3\beta^3 + n_p\frac{ps}{V} + n_r\frac{rs}{V})\}$$

$$\dot{r} = \frac{\rho S V^2 s I_x}{I_x I_z - I_{xz}^2}\{ n_1\beta + n_3\beta^3 + n_p\frac{ps}{V} + n_r\frac{rs}{V} \tag{4.1c}$$

$$+ \frac{I_{xz}}{I_x}(l_1\beta + l_3\beta^3 + l_p\frac{ps}{V} + l_r\frac{rs}{V})\}$$

$$\dot{\phi} = p + \tan\alpha\, r \tag{4.1d}$$

Here, α is the (assumed constant) angle of attack, β and ϕ are the angles of sideslip and roll, respectively, and p and r are the rates of roll and yaw, respectively. The reader is referred to [6, 20] for definitions of the parameters appearing in (4.1), though we note that aerodynamic coefficients cubic in the sideslip angle β appear, with various parameters depending as well on the angle of attack α. Precise forms of these dependencies are given by Ross [6] , which are derived by approximating flight test data. Specifically, for a particular configuration and flight condition, we have

$$y_p = 0.014 + 0.505\alpha - 0.47\alpha^2 \tag{4.2a}$$

$$l_p = -0.132 + 0.08\alpha \tag{4.2b}$$

$$n_p = 0.0125\alpha - 0.938\alpha^2 \tag{4.2c}$$

$$y_r = 0 \tag{4.2d}$$

$$l_r = 0.006 + 0.54\alpha \tag{4.2e}$$

$$n_r = -0.351 - 0.089\alpha \tag{4.2f}$$

Other (nonaerodynamic) parameters are set as follows [6] : $\rho = 0.906$ kg/m^3, $V = 71.25$ m/sec, m $= 2154$ kg, $s = 3.05$ m, $S = 40.18$ m^2, $I_{zz} = 2182$ kg m^2, $I_{xx} = 25430$ kg m^2, $I_{xz} = 1615$ kg m^2.

The parameters which remain to be specified in Eq. (4.1) are the linear and cubic coefficients in the aerodynamic moments in sideslip, roll and yaw. These are y_1, y_3, l_1, l_3, and n_1, n_3, respectively. The Hopf bifurcation analysis package BIFOR2 [16] was used to numerically investigate the influence of these parameters on the stability of a Hopf bifurcation in the dynamics. The results showed a strong dependence of the

stability coefficient β_2 on the nonlinear yaw parameter n_3. The values of these parameters obtained from [6, 20] are: $y_1 = -0.191$, $y_3 = -1.958$, $l_1 = -0.184$, $l_3 = -0.0$, $n_1 = 0.07$, $n_3 = 2.65$. In particular, we note that from Figure 6 of [6] , the values of n_1 and n_3 are positive for the nominal configuration. Thus, the negative values for these parameters quoted in [20] do not appear to agree with the actual data. It is therefore not surprising that the calculations we have performed do not coincide exactly to those of [20] . Note also that we have followed [6] in assuming an air density value of $\rho = 0.906$, while a value of 0.5 is quoted in [20] .

For the nominal parameter values given in the foregoing, the program BIFOR2 computes a Hopf bifurcation from the origin for Eq. (4.1) with a critical angle of attack $\alpha_c = 0.295$ rad $= 16.9$ deg, in basic agreement with the references. The bifurcation is supercritical, with the bifurcation stability coefficient $\beta_2 = -4.1$. The calculations for the nominal system also reveal a difficulty with the assumed aerodynamic model: the eigenvalues cross the imaginary axis from the right half of the complex plane into the left half plane as α is increased through its critical value. For the nominal values above, BIFOR2 determines that the rate of change of the real part of these eigenvalues with respect to α is -0.937. The noncritical eigenvalues of (4.1) at the bifurcation point are found to be -0.64 and -0.05, which compare well with [20] when the differences in the parameter values used are taken into consideration.

In order to illustrate application of the bifurcation control results discussed in Section 3, we consider the effect of the parameter n_3 on stability of the Hopf bifurcation. Using a formula for β_2 given in [3] , we have determined that β_2 for this problem depends *linearly* on the coefficient n_3. This fact is easily obtained since only cubic nonlinearities in one component of the state (β) appear in (4.1). In particular, we find that *increasing* the value of n_3 from 2.65 results in a more negative stability coefficient β_2, and hence in better local stability properties. In contrast, decreasing n_3 increases β_2, possibly leading to a subcritical (unstable) bifurcation. For example, taking $n_3 = -10$ results in $\beta_2 = 2.56$, and hence an unstable bifurcation.

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